$y = \sqrt{x} \quad ; \quad x = 1, x = 4$ and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

$V = \int_{1}^{4} 2\pi x\sqrt{x}dx$

▶

$V = \int_{1}^{4} 2x\sqrt{x}dx$

▶

$V = \int_{0}^{4} 2x\sqrt{x}dx$

▶

$V = \int_{-4}^{4} 2x\sqrt{x}dx$

▶

(Marks: 1) - Please choose one

Let f is a smooth curve on the interval [a, b]. What is the arc length L of the curve f(x) defined over the interval [a, b]?

$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))}$

▶

$L = \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))}\Delta x_k$

▶

$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))^2}\Delta x_k$

▶

$L = \sum_{k=1}^{n} \sqrt{1 + (f(x^*_k))}\Delta x$

▶

(Marks: 1) - Please choose one

Let $f(x)$ is a function such that as x approaches a real number a, either from left or right-hand-side, the function values increases or decreases unboundedly then

$\lim_{x \to a} f(x)$

- ▶ Exist
- ▶ Does not exist

(Marks: 1) - Please choose one

$\frac{d(\sec x)}{dx} =$

- ▶ $(\sec x)(\tan x)$
- ▶ $(\sec x)(\tan x)$
- ▶ $(co\sec x)(co\,t\,x)$
- ▶ $(co\sec x)(\tan x)$

(Marks: 1) - Please choose one

At what points the two curves: $y = x^2 \; and \; y = x + 6$ intersect ?

- ▶ $x = 0 \; and \; x = 2$
- ▶ $x = 0 \; and \; x = 3$
- ▶ $x = 2 \; and \; x = 3$
- ▶ $x = -2 \; and \; x = 3$

(Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x \to -\infty} f(x) = +\infty \;\; and \;\; \lim_{x \to +\infty} f(x) = +\infty$ then f has _____ on $(-\infty, +\infty)$

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

For a graph to be symmetric about y-axis means, for each point (x,y) on the graph,the point ---------- is also on the graph

► (x , -y)
► (-x , y)
► (-x , -y)

(Marks: 1) - Please choose one

The graph $x = y^2$ is symmetric about ---------axis

► X-axis
► Y-axis
► Origin

(Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as **:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

(Marks: 1) - Please choose one

$$a_n = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

Which of the following option is true for the sequence ?

► Increasing
► Decreasing
► Nonincreasing
► Nondecreasing

(Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be**:**

► Divergent
► Convergent
► Give no information

(Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... \ \ where\ (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

(Marks: 1) - Please choose one

If $\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k}$ where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent
► Divergent
► Give no information

(Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is ..... of x

► relation
► function
► not a function
► not a relation

(Marks: 1) - Please choose one

Domain of the function y = $\frac{(x^2 - 4)}{(x - 2)}$ is

► $(-\infty, 2) U (2, +\infty)$
► $(-\infty, 2)$

▶ $(-\infty, 0)$

(Marks: 1) - Please choose one

Tan(x) is continuous every where except at points

▶ $\pm\frac{k\pi}{2}(k = 1,3,5,...)$

▶ $\pm\frac{k\pi}{2}(k = 2,4,6,...)$

▶ $\pm\frac{k\pi}{2}(k = 1,2,3,4,5,6,...)$

(Marks: 1) - Please choose one

$\lim_{x\to 0}\frac{\sin x}{x}$

= ----------

▶ -1
▶ 2
▶ 0
▶ 1

(Marks: 1) - Please choose one

How the series $1-3+5-7+9-11$ can be expressed in sigma notation?

▶ $\sum_{k=0}^{k=5}(-1)^k(2k+1)$

▶ $\sum_{k=1}^{k=5}(-1)^k(2k+1)$

$$\sum_{k=1}^{k=5}(2k+1)$$

▶

$$\sum_{k=1}^{k=5}(2k+1)$$

▶

(Marks: 1) - Please choose one

What is the sum of following series?

$1^3+2^3+3^3+4^3+\_\_\_\_\_+n^3$

$$\frac{n(2n)(2n+1)}{6}$$

▶

$$\frac{(n+1)(n+2)}{2}$$

▶

$$\left[\frac{n(n+2)}{2}\right]^2$$

▶

$$\left[\frac{n(n+1)}{2}\right]^2$$

▶

(Marks: 1) - Please choose one

$\frac{5}{7}\times1^2+\frac{5}{7}\times2^2+\frac{5}{7}\times3^2+\frac{5}{7}\times4^2....+\frac{5}{7}\times n^2=$ _____

$$\frac{5n(n+1)(2n+1)}{42}$$

▶

$$\frac{5n(n+1)}{14}$$

▶

$\frac{5n^2(n+1)^2}{14}$

▶

$\frac{5(n+1)(2n+1)}{42}$

▶

(Marks: 1) - Please choose one

$\int_a^a f(x)dx =$ _____

If point a is in the domain of function f, then

▶ $f'(x)$

▶ $f(x)$

▶ 0

▶ 1

(Marks: 1) - Please choose one

If $a_1 > a_2 > a_3 > ..... > a_n > ....$, then a sequence $\{a_n\}$ is .....

▶ Increasing

▶ Nondecreasing

▶ Decreasing

▶ Nonincreasing

(Marks: 1) - Please choose one

A series of the form $\sum_{k=1}^{\infty}(-1)^n a_k$ is called _____.

▶ Alternating series

▶ Geometric series

▶ Arithmetic series

▶ Harmonic series

(Marks: 1) - Please choose one

Which of the following is the Maclaurin series for e^x ?

$$1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$1+x+\frac{x^3}{3!}+\ldots+\frac{x^k}{k!}+\ldots$$

▶

$$1-x+\frac{x^3}{3!}-\ldots-\frac{x^k}{k!}-\ldots$$

▶

(Marks: 1) - Please choose one

Which of the following is the work done *W* if an object moves in the positive direction along a coordinate line while subject to a force *F(x)* in the direction of motion over an interval [0,3]?

$$W=\int_{2}^{3}3xdx$$

▶

$$W=\int_{0}^{3}3xdx$$

▶

$$W=\int_{0}^{3}F(x)dx$$

▶

$$W=\int_{3}^{0}F(x)dx$$

▶

(Marks: 1) - Please choose one

Which of the following is the spring constant *k* if a spring whose natural length is 2*m* exerts a force of 3*N* when stretched 1*m* beyond its natural length?

▶ 3 *x*

▶ 3 *N/m*

▶ 2 *m*

► 3 *m/N*

(Marks: 2)

Find the limits of the integral indicating the area bounded by the curves $y = x^2 \; and \; y = x + 6$.

(Marks: 2)

What will be the amount of work done if an object moves 7*m* in the direction of a force of 70*N*?

(Marks: 2)

Evaluate the following integral by substitution method.

$$\int x\,(2x^2+1)^{\frac{2}{3}}\,dx$$

(Marks: 3)

Evaluate the following integral:

$$\int \frac{5-6\sin^2 x}{\sin^2 x}\,dx$$

(Marks: 3)

Find the spring constant '*k*'; if a force of 10*N* is required to stretch a spring from its natural length of 4.8*m* to a length of 6.8*m*?

(Marks: 3)

Find a definite integral indicating the area of the surface generated by revolving the curve $y = \sqrt[3]{3x} \;\; ; \;\; 0 \le y \le 4$ about the $x-$ axis. But do not evaluate the integral.

(Marks: 5)

Let $\frac{d}{dx}\left[f(x)\right] = 12x^2 - 6x + 1$. Find $f(x)$

(Marks: 5)

Determine whether the sequence $\{a_n\}$ converges or diverges; if it converges then find its limit;

$$a_n = \frac{3n^4+1}{4n^2-1}$$

where

(Marks: 5)

Use the cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y = x^3 , x = 1, y = 0$ is revolved about the *y*-axis.

(Marks: 10)

Find the area of the region that is enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ between $x = \frac{1}{4}$ and $x = 1$.

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ..... At x_0

► Minima
► Maxima
► None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ..... At x_0

► Minima
► Maxima
► None of these

Question No: 3 (Marks: 1) - Please choose one

$\lim_{x \to 0} \frac{\sin 2x}{x} = - - - - -$

► 2
► 4

- 1
- ∞

Question No: 4 (Marks: 1) - Please choose one

$$\lim_{x\to 0^+} \frac{\ln x}{1/x} = -----$$

- 1
- 0
- e
- None of these

Question No: 5 (Marks: 1) - Please choose one

$$\frac{d(\tan x)}{dx} =$$

- $\sec x$
- $\sec^2 x$
- $co\sec x$
- $co\sec^2 x$

Question No: 6 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- 0
- $\frac{-1}{x^2}$
- $\frac{4}{x^2}$
- $\frac{-4}{x^2}$

Question No: 7 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}\left((c)\{h(x)\}\right) =$ _____

- 0
- $\frac{d}{dx}\left(h(x)\right)$
- $\frac{d}{dx}\left(h(cx)\right)$
- $c\frac{d}{dx}\left(h(x)\right)$

Question No: 8 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$\frac{d}{dx}[f][g] =$

- $\frac{[f'][g]-[f][g']}{g^2}$
- $[f'][g']$
- $[f'][g]+[f][g']$
- $[f'][g]-[f][g']$

Question No: 9 (Marks: 1) - Please choose one

The power rule, $\frac{d}{dx}[x^n] = nx^{n-1}$ holds if n is _____

- An integer
- A rational number
- An irrational number
- All of the above

Question No: 10 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then

which of the following statement is correct?

- ▶ f is a decreasing function
- ▶ f is an increasing function
- ▶ f is a constant function

Question No: 11 (Marks: 1) - Please choose one

If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- ▶ f is concave up on (a, b).
- ▶ f is concave down on (a, b)
- ▶ f is linear on (a, b).

Question No: 12 (Marks: 1) - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

What does 'n' represent in Riemann Sum ?

- ▶ No. of Circles
- ▶ No. of Rectangles
- ▶ No. of Loops
- ▶ No. of Squares

Question No: 13 (Marks: 1) - Please choose one

$$\lim_{x\to-\infty} f(x) = +\infty \quad and \quad \lim_{x\to+\infty} f(x) = +\infty$$

If f is continuous function such that $(-\infty, +\infty)$ then f has _____ on

- ▶ maximum value but no minimum
- ▶ minimum value but no maximum
- ▶ both maximum and minimum value

Question No: 14 (Marks: 1) - Please choose one

$$\int_2^t \frac{x^2}{2} dx$$

The expression , represents a function of :

- ▶ t
- ▶ x
- ▶ 2
- ▶ Both t and x

Question No: 15 (Marks: 1) - Please choose one

$\int cf(x)dx = \_\_\_\_\_$

if c is a constant

- 0
- c
- $\int f(cx)dx$
- $c\int f(x)dx$

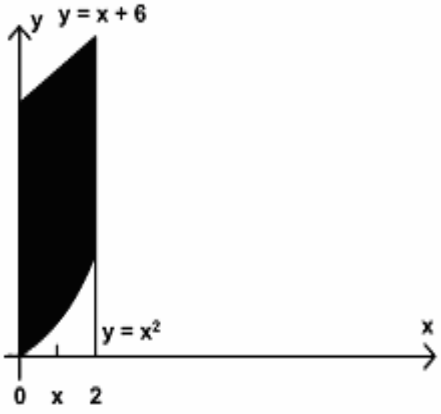
Question No: 16 (Marks: 1) - Please choose one

Sigma notation is represented by which of the following Greek letter?

- χ
- η
- Σ
- ψ

Question No: 17 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**

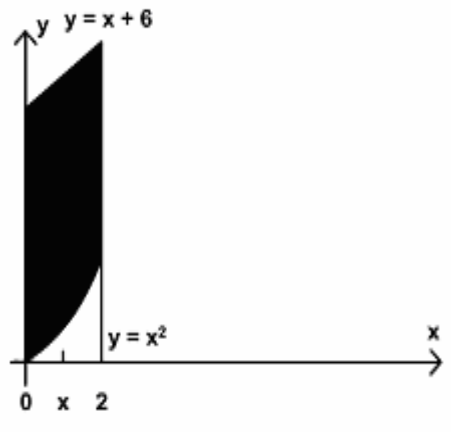


- $y = x + 6$
- $y = x^2$
- $x = 2$

- $x = 0$

Question No: 18 (Marks: 1) - Please choose one

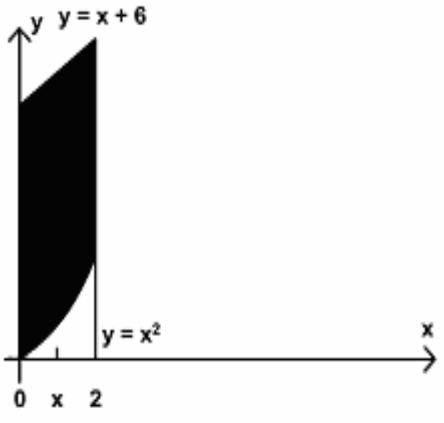
In the following figure, the area bounded on the sides by the lines are **:**



- $x = 0$
- $x = 2$
- $x = 0 \; and \; x = 2$
- $x = 6$

Question No: 19 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



- $A = \int_{0}^{2} \left[(x+6) - \left(x^2\right) \right] dx$
- $A = \int_{x}^{2} \left[(x+6) - \left(x^2\right) \right] dx$

$$A=\int_0^2\left[(x+6)+\left(x^2\right)\right]dx$$

▶

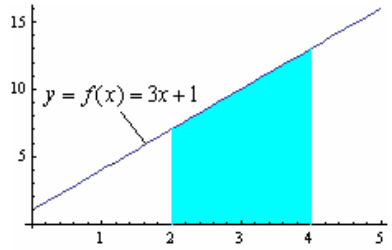
$$A=\int_0^x\left[(x+6)-\left(x^2\right)\right]dx$$

▶

Question No: 20 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y=f(x)=3x+1$ over the interval $[2, 4]$, evaluated by using the formula

$$Area=f(x_1^*)\,\Delta x+f(x_2^*)\,\Delta x$$



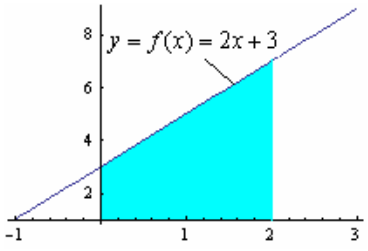
If the interval $[2, 4]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are left endpoint of each sub-interval.

▶ 17
▶ 20
▶ 23

Question No: 21 (Marks: 1) - Please choose one

Which of the following is approximate area under the curve $y=f(x)=2x+3$ over the interval $[0, 2]$, evaluated by using the formula

$$Area=f(x_1^*)\,\Delta x+f(x_2^*)\,\Delta x$$



If the interval $[0, 2]$ is divided into two sub-intervals of equal length and x_1^* and x_2^* are right endpoint of each sub-interval.

▶ 8
▶ 10
▶ 12

Question No: 22 (Marks: 1) - Please choose one

$If\ \ x>0\ then\ \frac{d}{dx}[\ln x]\ =$ _____

► 1

► x

1

► x

$\ln x$ 1

►

Question No: 23 (Marks: 1) - Please choose one

Suppose f and g are integrable functions on [a,b] and c is a constant, then $\int_a^b c\,[f(x)+g(x)]\,dx =$ _____

$\int_a^b f(cx)dx + \int_a^b g(cx)dx$

►

$\int_a^b f(x)\,dx + \int_a^b g(x)]\,dx$

►

$c\int_a^b f(x)dx + c\int_a^b g(x)dx$

►

► 0

Question No: 24 (Marks: 1) - Please choose one

If the function f is continuous on [a,b] and if $f(x) \geq 0$ for all x in [a,b], then which of the following gives area under the curve $y = f(x)$ over the interval [a,b]?

$\lim_{x\to\infty} \sum_{k=1}^{n} [x_k][f(x_k)]$ *where n is number of subdivisions of* $[a,b]$

►

$\int_a^b f(x)\,dx$

►

$\pi[radius]^2$

►

► (Width) (Height)

Question No: 25 (Marks: 1) - Please choose one

Let region R in the first quadrant enclosed between $y = 3x \quad and \quad y = 2x^2$ is revolved about the
x-axis .Which of the following equation gives the volume of a solid by cylindrical shells?

$V = \int_0^{\frac{3}{2}} 2\pi x(3x - 2x^2)dx$

▶

$V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$

▶

$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$

▶

$V = \int_{-1}^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$

▶

Question No: 26 (Marks: 1) - Please choose one

Let f is a smooth function on [a, b]. What will be the arc length L of the curve y = f(x) from x = a to x = b?

$L = \int_b^a \sqrt{1 + [f'(x)]}dy$

▶

$L = \int_a^b \sqrt{1 + [f'(x)]^2}dx$

▶

$$L=\int_{0}^{a}\sqrt{1+[f'(x)]}dy$$

►

$$L=\int_{a}^{b}\sqrt{1+[f'(x)]}dx$$

►

Question No: 27 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the

$$\int_{a}^{b}f(x)dx=\lim_{l\to a}\int_{l}^{b}f(x)dx$$

integral defined by is called **:**

► Improper
► Proper
► Line

Question No: 28 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n}>1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n}<1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 30 (Marks: 1) - Please choose one

$$\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx$$

Consider the indefinite integral

Let $t = x^3+2x^2+x-3$

Is the following substitution correct?

$$\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx = \int \frac{1}{t}\,dt$$

- ► Yes
- ► No

Question No: 31 (Marks: 1) - Please choose one

$$\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$$

The series $\sum u_k$ be a series with positive terms and suppose that if $\rho = 1$, then which of the following is true?

- ► Converges
- ► Diverges
- ► May converges or diverges
- ► Gives no information

Question No: 32 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

- ► Converges
- ► Diverges
- ► May converges or diverges
- ► Gives no information

Question No: 33 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} |u_k| = |u_1| + |u_2| + |u_3| + \ldots |u_k| + \ldots$$

If the series converges , then which of

$$\sum_{k=1}^{\infty} u_k = u_1 + u_2 + u_2 + \ldots u_k + \ldots$$

the following is true for ?

- ► Converges
- ► Diverges
- ► Gives no information

Question No: 34 (Marks: 1) - Please choose one

$$\rho = \lim_{k\to\infty} \frac{|u_{k+1}|}{|u_k|}$$

Let $\sum u_k$ be a series with nonzero terms and suppose that if $\rho = +\infty$, then which of the following is true?

- ► Then the series $\sum u_k$ diverges
- ► The series $\sum u_k$ converges absolutely and therefore converges
- ► May converges or diverges
- ► Gives no information

Question No: 35 (Marks: 1) - Please choose one

$\int_{-1}^{1} (x-1)\, dx =$ _____

- ► -2
- ► 0
- ► 2
- ► 4

Question No: 36 (Marks: 1) - Please choose one

How many critical points exist for a function f if

$f'(x) = (x-3)(x-2)$

- Zero
- One
- Two
- Four

Question No: 37 (Marks: 1) - Please choose one

$\log_b ac =$ _____

- $\log_b a + \log_b c$
- $\log_b a - \log_b c$
- $\frac{\log_b a}{\log_b c}$
- $(\log_b a)(\log_b c)$

Question No: 38 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

- $a \log_b r$
- $r \log_b a$
- $\frac{\log_b a}{\log_b r}$
- $\log_b a + \log_b r$

Question No: 39 (Marks: 1) - Please choose one

Let $y = \frac{2\sqrt{2}}{3} x^{\frac{3}{2}}$; $0 \le x \le 2$ then which of the following is the length of the curve?

$$L=\int_0^2\sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int_0^2\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\,dx$$

▶

$$L=\int_0^2\sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]}\,dx$$

▶

Question No: 40 (Marks: 1) - Please choose one

Which of the following are *first two* terms for the Taylor series of $f(x)=e^{-x}$ at $x = 0$?

$1+(1)(x-0)$

▶

$1+(-1)(x+0)$

▶

$1+(-1)(x-0)$

▶

$(-1)(x-0)$

▶

Question No: 41 (Marks: 2)

$$\int_2^3(1-x)dx$$

Evaluate the integral

$$\int_2^3 (1-x)dx$$

$$= \left| x - x^2 \frac{1}{2} \right|_2^3$$

$$= \frac{1}{2} \left| 2x - x^2 \right|_2^3$$

$$= \frac{1}{2}(2(3-2)-(3-2)^2)$$

$$= \frac{1}{2}(2-1)$$

$$= \frac{1}{2}$$

Question No: 42 (Marks: 2)

$$\int_2^{+\infty} \frac{dx}{x^2}$$

Evaluate the improper integral

Question No: 43 (Marks: 2)

A function $f(x) = x^2 - 4x - 9$ has critical point 2 in an interval [0, 5]. Find the maximum value of the function and point having this value.

Question No: 44 (Marks: 3)

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Evaluate:

$$\int \frac{5 - 6\sin^2 x}{\sin^2 x} dx$$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2 \quad , \quad x > 0$, and bounded on the sides by the lines $y = 1$ and $y = 4$

$y = x^2 \quad , \quad x > 0$

So we have

$$A = \int_1^4 x^2 dx$$
$$= \left| \frac{x^3}{3} \right|_1^4$$
$$= \frac{1}{3}(4-1)^3$$
$$= \frac{1}{3}(3)^3$$
$$= 9$$

Question No: 46 (Marks: 3)

Determine whether the following sequence converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \frac{5n^2 - 1}{20n + 7n^2}$$

Question No: 47 (Marks: 5)

Use the Alternating series Test to determine whether the given series converges

$$\sum_1^\infty \frac{(-1)^{n-1} \cdot n!}{2^n}$$

Question No: 48 (Marks: 5)

Evaluate the integral

$$\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2} dt$$

Solution

$$\int_{\frac{\pi}{2}}^{0} \frac{1+\cos 2t}{2} dt$$

$u = 2t$

$$\frac{du}{dt} = 2dt$$

$du = 2dt$

so

$$= \frac{1}{4}\int_{\frac{\pi}{2}}^{0} 1+\cos u du$$

$$= \frac{1}{4} | u + \sin u |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4} | 2t + \sin 2t |_{\frac{\pi}{2}}^{0}$$

$$= \frac{1}{4}(2\frac{\pi}{2} + \sin 2\frac{\pi}{2})$$

$$= \frac{1}{4}(\pi + \sin \pi)$$

$$= \frac{1}{4}(\pi + 0)$$

$$= \frac{\pi}{4}$$

Question No: 49 (Marks: 5)

Evaluate the sums

$$\sum_{k=1}^{5} k(3k+5)$$

$$= 1(3+5) + 2(6+5) + 3(9+5) + 4(12+5) + 5(15+5)$$

$$= 8 + 22 + 3(45) + 4(60) + 5(75)$$

$$= 8 + 22 + 135 + 240 + 375$$

$$= 780$$

Question No: 50 (Marks: 10)

Find the volume of the solid that results when the region enclosed by the given curves is revolved about the x – axis**.**

$y = 1 + x^3$, $x = 1, x = 2, y = 0$

from $V = \int_a^b \pi[f(x)]^2 dx$

$$V = \int_1^2 \pi[1 + x^3]^2 dx$$

$$V = \int_1^2 \pi[1 + x^5 + 2x^3] dx$$

$$V = \pi \int_1^2 (1 + x^5 + 2x^3) dx$$

$$V = \pi \,|\, (x + \frac{1}{6}x^6 + \frac{1}{2}x^4) \,|_1^2$$

$$V = \pi((2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4)$$

$$V = \pi\{((2-1) + \frac{1}{6}(2-1)^6 + \frac{1}{2}(2-1)^4)\}$$

$$V = \pi((1) + \frac{1}{6} + \frac{1}{2}))$$

$$V = \frac{\pi(6+1+3)}{6}$$

$$V = \frac{\pi(10)}{6} = \pi\frac{5}{3}$$

This paper is solved by our best knowledge. In the case of any error/correction/suggestion, please contact at gulshanvu@yahoo.com, with reference to the concerned paper's number.

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ……. At x_0

► Minima
► Maxima
► None of these

Question No: 2 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) < 0$ then f has relative ……. At x_0

► Minima
► Maxima
► None of these

Question No: 3 (Marks: 1) - Please choose one

A line $x = x_0$ is called ---------- for the graph of a function f if $f(x) \to +\infty \; or \; f(x) \to -\infty$ as x approaches x_0 from the right or from the left

► Horizontal asymptotes
► None of these
► Vertical asymptotes

Question No: 4 (Marks: 1) - Please choose one

A line $y = y_0$ is called a ……. for the graph f if

$$\lim_{x\to+\infty} f(x) = y_0 \quad or \quad \lim_{x\to-\infty} f(x) = y_0$$

- ▶ Vertical asymptotes
- ▶ Horizontal asymptotes
- ▶ None of these

Question No: 5 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then $\frac{d}{dx}[x^n] =$

- ▶ x^{n-1}
- ▶ $n\,x^{n-1}$
- ▶ $n\,x^{n+1}$
- ▶ $(n-1)x^{n+1}$

Question No: 6 (Marks: 1) - Please choose one

If $f(x) = 3x^8 + 2x + 1$ then $f'(x) =$ _____

- ▶ $3x^7 + 2$
- ▶ $24x^7 + 2$
- ▶ $3x^9 + 2x^2$
- ▶ $24x^9 + 2x^2$

Question No: 7 (Marks: 1) - Please choose one

$\frac{d(\tan\ x)}{dx} =$

- ▶ $\sec x$
- ▶ $\sec^2 x$
- ▶ $co\sec x$
- ▶ $co\sec^2 x$

Question No: 8 (Marks: 1) - Please choose one

If $y = \frac{1}{1-x}$ then $\frac{dy}{dx} =$

- 1
- -1
- $\frac{1}{(1-x)^2}$
- $\frac{-1}{(1-x)^2}$

Question No: 9 (Marks: 1) - Please choose one

If $xy = 4$ then $\frac{dy}{dx} =$

- 0
- $\frac{-1}{x^2}$
- $\frac{4}{x^2}$
- $\frac{-4}{x^2}$

Question No: 10 (Marks: 1) - Please choose one

If $x^2 + y^2 = 9$ then $\frac{dy}{dx} =$

- $\frac{x}{y}$
- $\frac{-x}{y}$
-

▶ $\frac{-y}{x}$

▶ $\frac{y}{x}$

Question No: 11 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}\left((c)\{h(x)\}\right) =$ _____

▶ 0

▶ $\frac{d}{dx}\left(h(x)\right)$

▶ $\frac{d}{dx}\left(h(cx)\right)$

▶ $c\frac{d}{dx}\left(h(x)\right)$

Question No: 12 (Marks: 1) - Please choose one

Suppose that f and g are differentiable functions of x then

$\frac{d}{dx}[f][g] =$

▶ $\frac{[f'][g]-[f][g']}{g^2}$

▶ $[f'][g']$

▶ $[f'][g]+[f][g']$

▶ $[f'][g]-[f][g']$

Question No: 13 (Marks: 1) - Please choose one

$\frac{d}{dx}[\text{cosec } x] =$ _____

$\frac{-\cos x}{1-\cos^2 x}$

▶ $\frac{-\cos x}{1-\cos^2 x}$

$\frac{-\cos x}{1-\cos^2 x}$

▶

$\frac{1}{1-\cos^2 x}$

▶

Question No: 14 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point g(x), then the _____ is differentiable at point x .

▶ Composition (f o g)
▶ Quotient (f / g)
▶ Product (f . g)
▶ Sum (f + g)

Question No: 15 (Marks: 1) - Please choose one

$y = f\left(g\left(h(x)\right)\right)$

If

$u = g\left(h(x)\right)$

$v = h(x)$ then $\frac{dy}{dx} =$ _____

▶ $\frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$

$\frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}$

▶

$\frac{dv}{du}.\frac{du}{dv}.\frac{dy}{dx}$

▶

Question No: 16 (Marks: 1) - Please choose one

Chain rule is a rule for differentiating _____ of functions.

▶ Product
▶ Sum
▶ Difference
▶ Composition

Question No: 17 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denote points in that interval. If $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ then which of the following statement is correct?

▶ f is an increasing function.
▶ f is a decreasing function.
▶ f is a constant function.

Question No: 18 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denotes two distinct points in that interval. If $f(x_1) = f(x_2)$ for all points x_1 and x_2 then which of the following statement is correct?

▶ f is a decreasing function
▶ f is an increasing function
▶ f is a constant function

Question No: 19 (Marks: 1) - Please choose one

If $f''(x) > 0$ on an open interval (a,b), then which of the following statement is correct?

▶ f is concave up on (a, b).
▶ f is concave down on (a, b).
▶ f is linear on (a, b).

Question No: 20 (Marks: 1) - Please choose one

If $x > 0$ *then* $\frac{d}{dx}[\ln x] =$ _____

- 1
- x
- $\frac{1}{x}$
- $\ln \frac{1}{x}$

Question No: 21 (Marks: 1) - Please choose one

Let $y = (x^3 + 2x)^{37}$. Which of the following is correct?

$\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$

- $\frac{dy}{dx} = (37)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = 111x^2(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{36}$
- $\frac{dy}{dx} = (111x^2 + 74)(x^3 + 2x)^{38}$

Question No: 22 (Marks: 1) - Please choose one

What is the base of natural logarithm?

- 2.71
- 10

- 5
- Any real number

Question No: 23 (Marks: 1) - Please choose one

Let x_0 be critical points of the function f. Those critical points for which $f'(x_0) = 0$ are called _____ of f

- Local points
- End points
- Stationary points

Question No: 24 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

- $a \log_b r$
- $r \log_b a$
- $\frac{\log_b a}{\log_b r}$
- $\log_b a + \log_b r$

Question No: 25 (Marks: 3)

Discuss the concavity of the function $f(t) = (2-t)(t-3)+3$ on any interval using second derivative test?

For all constant numbers, derivative is zero.
So,
F'(t)=0

F''(t)=0…….derivative doesn't exist.

Question No: 26 (Marks: 5)

Find the derivative of the function $y = \ln(1+x^3)$

$$\frac{d}{dx}[\ln(1+x^3)] = \frac{1}{(1+x^3)}.\frac{dy}{dx}[1+x^3]$$

$$= \frac{1}{(1+x^3)}.3x$$

$$= \frac{3x}{(1+x^3)} \ldots Ans$$

Question No: 27 (Marks: 10)

Compute derivative of trigonometric function $Sin(x)$ by definition.

We want to know the derivative of $f(x) = \sin x$

so,

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)-\sin(x)+\cos(x)\sin(h)}{h}$$

$$= \lim_{h\to 0}[\sin(x)\left(\frac{\sin(h)}{h}\right)-\sin(x)\left(\frac{1-\cos(h)}{h}\right)]$$

As $\sin x$ *and* $\cos x$ *donot involve* h *so they will remain cons*$\tan t$ *here*

$$\lim_{\lim_{h\to 0}} \sin(x) = \sin(x)$$

$$\lim_{\lim_{h\to 0}} \cos(x) = \cos(x)$$

Now,

$$\frac{d}{dx}(\sin x) = \cos(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)-\sin(x)\lim_{h\to 0}\left(\frac{1-\cos(h)}{h}\right)$$

$$= \cos(x)(1)-\sin(x)(0) = \cos x$$

$$\frac{d}{dx}\sin(x) = \cos(x) \ldots\ldots\ldots proved$$